’Rebooting Communications’ - A Quantum-Domain Perspective

Soon Xin Ng, and Lajos Hanzo

***Abstract*—As we approach nano-scale integration on the wings of Moore’s Law, ’Quantum Engineering’ is becoming a buzzword, since at this scale signal processing is departing from the law of classical physics and enters the realms of quantum physics. We would still have the option of accommodating the ever more sophisticated signal processing solutions on larger chip areas without increasing the integration density, but the yield would be reduced and the chips begin to break up. Another alternative for the research community is to ’reboot communications’ and start the new era of ’Communications V 2.0’. This journey has to start with the understanding of some of the basic postulates of quantum physics - but you do not have to become a quantum physisist! Feynman suggested that instead of mapping the classical bits to 0 and 5 Volt, we map them for example to the spin or charge of an electron. The story unfolds by understanding this mapping operation, the transmission and reception of our new information bearer as well as the mitigation of the deleterious propagation, storage and signal processing effects. Since the resultant quantum information is much more sensitive to environmental impairments than the good old classical bits, sophisticated transmission and processing techniques have to be conceived under ’Communications 2.0.’ But as a benefit, perfectly secure communications becomes possible even in the face of malicious eavesdroppers.**

So, let the journey begin!

I. INTRODUCTION

The Internet has revolutionized our lives. This revolution was catalyzed by the groundbreaking disoveries of information theory, followed by the evolution of integrated circuit technology, which has broadly speaking followed the predictions of Moore’s Law ever since 1965. This trend has gradually led to nano-scale integration, where encountering quantum effects is no longer avoidable.

The processing of quantum-domain information has to obey the basic postulates of quantum physics, where a so-called *qubit* may be represented as the *superposition* of a logical one and a logical zero. More explicitly, we could visualize this superposition as a coin spinning in a box, hence being in an equi-probable superposition of ’head’ and ’tail’, so that we can avoid the somewhat unpalatable reference to the famous Schroedinger cat analogy. Metaphorically speaking, we have to carry out all quantum signal processing operations until the coin is spinning in the box, because once it has stopped, we can no longer ’manipulate’ or process it in the quantum-domain - it has ’collapsed’ back into the classical domain. Therefore upon lifting the lid of the box, we can reveal the classical-domain outcome, which is either ’head’ or ’tail’.

Another property of the above-mentioned qubits is that they must not be copied, because trying to copy them would results again in their collapse to the classical domain, hence precluding their further processing in the quantum domain. Instead, the so-called *entanglement* operation has to be used. Intriguingly, entangled qubits have the property that if we change the spin of the electron representing the qubit, that of its entangled pair is also changed at the same instant. However, this does NOT violate the speed of light, because the preparatory operations carried out before the entanglement is established have to obey propagation at the speed of light. Upon entangling large vectors of qubits representing the quantum-domain operands parallel processing becomes feasible, hence it becomes possible to construct so-called quantum computers capable of solving various classically intractable problems. Having said that, these bespoke quantum computers can still be outperformed in certain tasks by classical computers, but they are eminently suitable for taylor-made tasks, which cannot be efficiently solved by classical computers. In parallel to these alluring developments, next-generation systems aim for realizing flawless telepresence. It has also been predicted that the number of devices connected to the Internet will soon outnumber the entire human population of planet Earth [1]. In this context the power of superposition and entanglement may be harnessed for efficiently solving various problems, which have hitherto been deemed to be unsolvable in our lifetime.

A striking example demonstrating the power of quantum computing is Grover’s Quantum Search Algorithm (QSA), which is capable of finding a single solution in an unsorted database having N elements at a complexity order of $\mathcal{O}(\sqrt{N})$, whilst its classical full-search based counterpart requires on the order of $\mathcal{O}(N)$ cost-function evaluations.

As wonderful as it sounds, quantum computers also impose a massive threat on classical security and privacy. The most popular public cryptosystem known as RSA, heavily relies on the hardness of the so-called integer factorization problem. Although this problem is impractical to solve using the current classical computers, this will no longer be the case when a fully functioning quantum computer is available. For instance, the time required for breaking a 2048-bit public key can be reduced from billions of years - using classical computers - to a matter of minutes using a quantum computer [2].

Fortunately, quantum information processing also provides a wonderful solution for mitigating this emerging threat. Quantum key distribution (QKD) [3] constitutes one of the near-commercialized quantum technologies. QKD circumvents the problem of the impractical, but absolutely secure one-time pad secret key distribution of classical communication.

The authors are with the School of Electronics and Computer Science, University of Southampton, Southampton, SO17 1BJ, UK (email: {sxn, lh}@ecs.soton.ac.uk).

Therefore, QKD will remain provably secure in the face of the physical security attacks that may be carried out by quantum computers. Another impressive development has suggested that it is also possible to directly transmit classical information totally securely utilizing quantum channels, whilst relying on the so-called quantum secure direct communication (QSDC) protocol [6]–[8]. This field of finding a novel scheme for securely transmitting classical information using quantum-domain techniques is widely referred to as quantum cryptography.

At the time of writing quantum technologies gradually approach maturity, the exchange of quantum information will become inevitable and eventually ubiquitous. Connecting multiple quantum computers using quantum links potentially offers the capability of outperforming a single quantum computer by creating a larger distributed quantum computer. One of the key requirements for creating such a system is the capability to maintain seamless quantum links amongst the quantum computers. The vital resource required in this architecture is the so-called maximally entangled pair, which is also referred to as the Einstein-Podolski-Rosen (EPR) pair, potentially facilitating an instantaneous action at a distance. This entangled pair is created in a unique superposition state so that any operation applied to one of the particles will immediately affect the other particle, even of they are separated by a great distance - again, provided that the appropriate preparatory entanglement operation have been carried out.

As quantum technologies become more prevalent in mainstream publications, several questions have emerged concerning what quantum technologies can offer in the realms of communication engineering. Although we have touched upon them briefly, in this short article, we would like to highlight several promising applications of quantum engineering and communications to motivate further research.

II. QUANTUM-BASED COMMUNICATION

Again, in contrast to classical bits, which can only assume a value of “0” or “1” in any bit interval, a qubit can hold both values simultaneously in a form of superposition as shown in Fig. 1. Therefore, N qubits in a state of superposition can be used to hold all the 2^N classical bit combinations.

Another highly relevant property of quantum information in this context is the *no-cloning theorem*, which we have briefly alluded to above by stating that upon trying to copy the qubits they collapse to the classical domain. In scientific parlance this dictates that no unitary operation can perform a perfect copying operation of a qubit in an unknown superposition state to another qubit. These two properties, in addition to the entanglement, can be exploited for developing several novel communication protocols.

Quantum key distribution (QKD) [3] constitutes one of the most well-known applications of quantum communication, albeit in all truth QKD only represents a secret key negotiation protocal. By relying on the no-cloning theorem and the fact that the action of ’measurement’ or observation collapses the superposition of quantum states to the classical domain sharing a so-called ’one-time pad’ secret key now becomes plausible. The seminal QKD proposal is commonly referred to as the Bennett-Brassard protocol (BB84) [3], which is based on the so-called ’prepare-and-measure’ protocol, while the E91 protocol [4] is based on pre-shared entanglement.

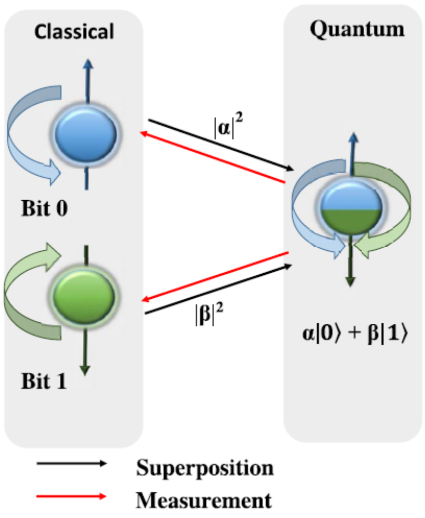


Fig. 1: A qubit can be in a superposition of two values or states at the same time. However, this superposition will collapse after measurement with a certain probability for each value “0” and “1”.

One of the features of a qubit is that it can be used to convey either quantum information or classical information. While the QKD protocol can be used for the exchange of the classical secret key, **quantum superdense coding** [5] supports the secure transmission of classical information through pre-shared EPR pair. This was an early demonstration that instead of acting as the medium of exchanging the secret key, the pre-shared entanglement can be used directly to transfer confidential classical information. This ingenious concept was then ultimately further developed by the proposal of **quantum secure direct-communication (QSDC)** [6], which constitutes a fully-fledged confidential quantum communications protocol, rather than being a pure secret key negotiation procedure. Given the increasing number of mobile devices communicating by broadcasting information, the secrecy and the privacy of the information becomes more crucial than ever. Quantum cryptography may pave the way for providing unbreachable physical layer security for next-generation communication. Naturally, there numerous open challenges in the way of widespread QSDC, such as its limited attainable rate and distance, as well as its reliance on quantum memory, which future research has to tackle.

To expound a little further, the direct transfer of quantum information over a quantum channel faces the following challenges. Firstly, due to the no-cloning theorem, any quantum information that is lost during its transmission cannot be readily replaced. Hence the traditional method of ensuring a reliable transmission by sending multiple copies of the same information is no longer feasible. However, the properties of quantum mechanics allow us to transfer quantum information without sending it through the quantum channel with the

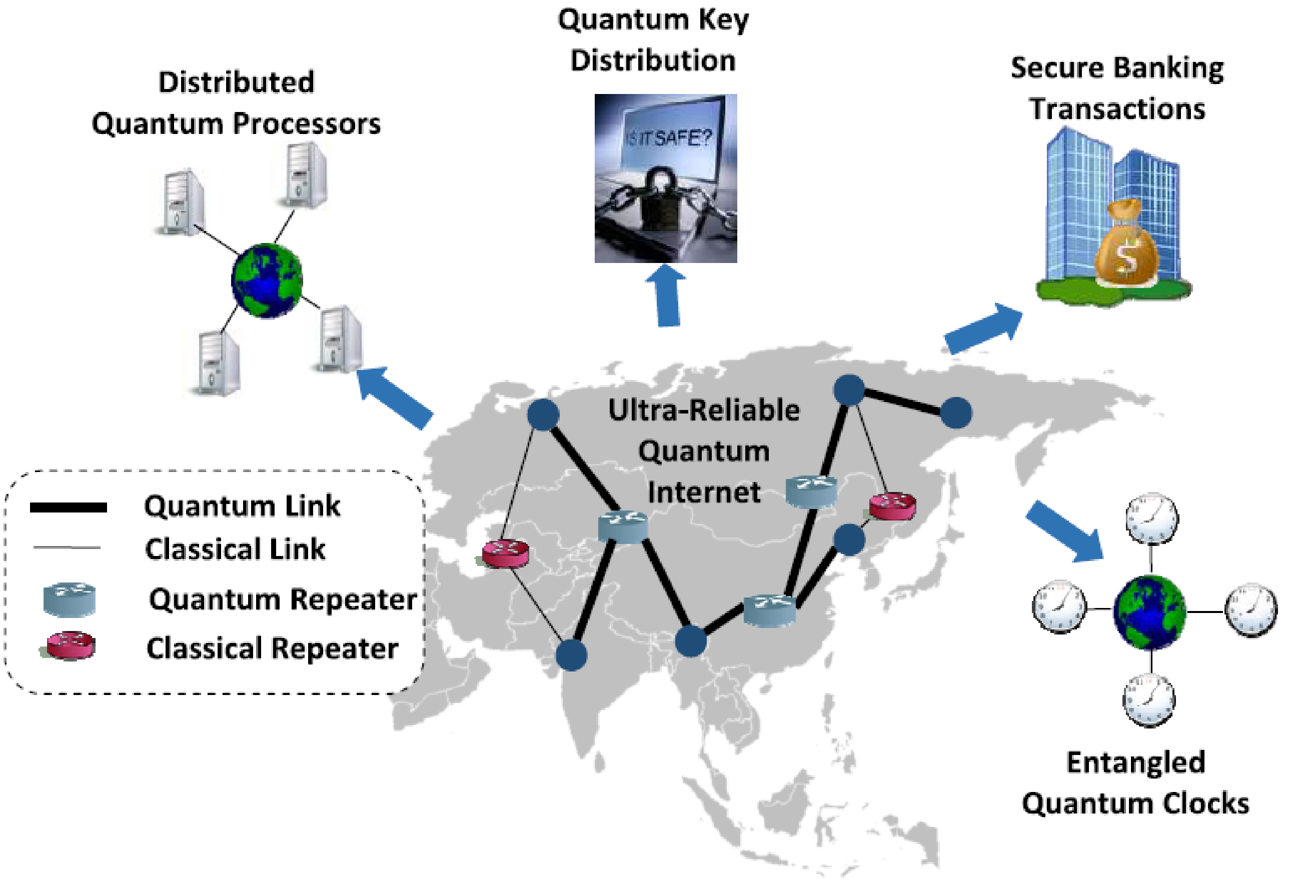


Fig. 2: Stylized vision of the quantum Internet of the near future, which will rely on a combination of both classical and quantum devices.

aid of **quantum teleportation** [9]. The transfer of quantum information can be replaced by the joint action of an EPR pair and classical communication. The employment of quantum teleportation is promising for several reasons. Firstly, multiple copies of EPR pairs can be generated, hence an error control procedure commonly referred to as ’distillation’ can be invoked. As a benefit, classical communication has a higher integrity than the direct transmission of quantum information.

Therefore, a paradigm shift is taking shape concerning the role of repeaters and network coding. For a quantum network, both **quantum repeaters** and **quantum network coding** [10], [11] are indispensible for the reliable distribution of the EPR pairs across multiple nodes in the context of long-distance transmissions. While in classical networks the operation of the repeater is often based on the decode-and-forward mechanism, in the quantum domain the role of the repeater is to maintain connectivity in the form of the seamless generation and sharing of EPR pairs between quantum nodes. To support this functionality, each quantum repeater may rely on the capability of performing *entanglement swapping* and *entanglement distillation*. This, in turn, will hinge on several novel network utilization metrics, which must be considered during the quantum network design of the near future.

The long-term goal in the exploration of quantum computation and communication is to conceive the perfectly secure **quantum Internet** [10], which is an emerging concept in the landscape of quantum engineering, as portrayed in the stylized illustrated of Fig. 2. The concept is reminiscent of that of the classical Internet, interconnecting multiple quantum nodes in the quantum network. The quantum Internet will facilitate the perfectly secure exchange of quantum information, whilst supporting a plethora of other compelling applications such as distributed quantum computation [12], blind quantum computation [13], quantum secret sharing [14], and many more. For example, multiple interconnected quantum computers can jointly act as a distributed quantum computer and can perform a more advanced computational tasks, than a single quantum computer. However, there are numerous other attractive applications that cannot even be predicted at the time of writing.

III. QUANTUM-SEARCH AIDED COMMUNICATIONS

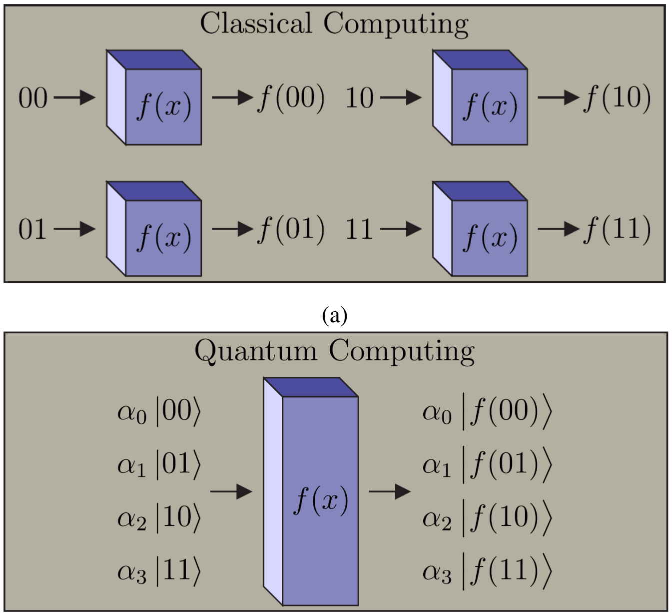
The inherent parallelism of quantum information processing intimated in Fig. 3 equips quantum computers with immense computational power. It has been shown theoretically that there are several classes of problems that can be solved very efficiently by quantum computers, such as integer factorization, finding solutions in large unstructured databases and large-scale optimization problems, just to name a few.

In this context the intriguing question is, how we exploit this beneficial computational speed-up to solve large-scale problems of classical communications. Hence, this section will be dedicated to the various applications of quantum computing algorithms, which have been shown to be capable of solving diverse problems arising in classical communication.

Quantum-Search Aided Multi-User Detection (QMUD) [15]. The high complexity of numerous optimal full-search based classical communication schemes, such as the maximum likelihood (ML) multiuser detector (MUD), often prevents their practical implementation. In this scenario, Grovers quantum search algorithm (QSA) may be invoked in the detection procedure, by exploiting its inherent parallelism for approaching the ML MUDs performance at a substantially reduced number of cost function evaluations. It succeeds in finding the solution after $\mathcal{O}(\sqrt{N})$ cost function (CF) evaluations, in contrast to the optimal classic full-search algorithms that require $\mathcal{O}(N)$ CF evaluations.

Quantum-Search Aided Multi-objective Routing [16]. The emergence of the Internet of things (IoT), as well as the research of next-generation wireless systems have motivated the development of self-organizing networks (SONs). They can act autonomously for the sake of achieving the best possible performance. The associated routing protocols have to strike a delicate trade-off amongst a range of conflicting quality-of-service (QoS) requirements. Finding the optimal solution typically becomes a non-polynomial-hard problem, as the network size increases in terms of the number of nodes. Moreover, the employment of user-defined utility functions often leads to suboptimal solutions. The concept of Pareto optimality comes to rescue, which is capable of amalgamating conflicting design objectives. In this context the Pareto front represents the collection of all optimal solutions, where none of the metric in the objective function can be improved without degrading at least one of the others, as exemplified by the BER vs. transmit power trade-off, just to mention one of them. Although there are a plethora of bioinspired algorithms suitable for solving this optimization problem, they often fail to generate all the optimal solutions constituting the optimal Pareto front. As a remedy, a quantum-aided multi-objective optimization algorithm can be constructed, which is capable of finding all Pareto-optimal routes at a reduced complexity. As a result, the complexity of finding the best route can be reduced to the order of $\mathcal{O}(N)$ and $\mathcal{O}(N\sqrt{N})$ in the best- and the worst-case scenarios, respectively. This corresponds to a substantial complexity reduction from the order of $\mathcal{O}(N^2)$ imposed by the brute-force full-search method.

Quantum-Search Aided Non-coherent Detection [17]. In large-dimensional wireless systems, such as cooperative multicell processing, millimeter wave, and massive multiple input multiple output systems, or cells having a high user density, such as airports, train stations, and metropolitan areas accurate estimation of all the channel gains is required for performing coherent detection. However, every time the Doppler frequency is doubled, the pilot overhead used for sampling the channel's complex-valued envelope also has to be doubled. Therefore, both the pilot overhead as well as the complexity escalate at high Doppler frequencies. As an attractive design alternative, differential modulation relying on noncoherent detection may be invoked for eliminating the pilot overhead, albeit at the cost of some performance degradation. As a beneficial solution, quantum-search assisted multiple symbol differential detection may be employed for matching the performance of the optimal full-search-based multiple symbol differential detectors, despite requiring a significantly reduced number of cost-function evaluations.



(a)

(b)

Fig. 3: The comparison of classical and quantum computation. The inherent parallelism of quantum information may provide a quantum computational speed-up for some classes of problems.

Joint Quantum-Search Aided Channel Estimation and Data Detection [18]. Joint channel estimation and multi-user detection (MUD) is capable of approaching the performance of a perfect channel estimation by iteratively exchanging soft extrinsic information between these two components of the receiver. It was demosntrated in this treatise that a quantum-assisted repeated weighted boosting search (QRWBS) algorithm may be readily combined with a quantum-search assisted MUD (QMUD) for iterative channel estimation and data detection in the uplink of multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing systems. This powerful systems is capable of operating in rank-deficient systems, where the number of receive antenna elements (AEs) at the base station (BS) is lower than the number of users transmittng in the uplink. It was also shown that QRWBS-aided channel estimation is capable of outperforming its classical counterpart, despite requiring a lower number of cost-function evaluations, which is an explicit benefit of invoking iterations between the MUD, the channel estimator and the channel decoders at the BS's receiver.

Quantum-Search Aided Localization [19]. With the proliferation of millimeter-Wave (mm-Wave) systems and visible light communications (VLCs), indoor localization may find multiple beneficial applications. When high localization accuracy is required and triangulation is not possible due to the infrastructure and scenario limitations, the computational complexity of carrying out a full search on the finely-grained grid of all possible tiles of the search area may become excessive. In this scenario a quantum search algorithm may come to rescue for reducing the computational complexity required for achieving the optimal full-search-based performance.

Suffice to say in conclusion of this scetion that many more attractive applications can be found in the literature and some others are yet to be discovered. Quantum technology has opened new avenues for solving problems that previously were impossible to solve. This gives us the perfect timing to revisit the hitherto unsolved problems of classical signal processing and communications domain and check, whether quantum-aided solutions might provide the long-awaited answers.

IV. QUANTUM DECOHERENCE

The most grave challenge of quantum signal processing is how to mitigate the deleterious effects of quantum decoherence [10], which inevitably affects the results of quantum computation or communication tasks - just like the Brownian motion of electrons imposes ubiquitous Gaussian noise in the classical receivers. Completely isolating the qubits from any environmental influence is practically impossible, hence the mitigation of these effects is paramount.

The employment of quantum error correction codes (QECC) is one of the most potent design alternatives of mitigating the decoherence. Even though error correction has been shown to perform well in the classical domain, implementing the QECCs imposes its own challenges. Indeed, any error correction procedure, both classical and quantum, depends on attaching redundancy to the information, which will be invoked at the decoder for error correction. In the classical domain, the effect of noise in the encoder and decoder circuitry may be deemed negligible in comparison to the noise inflicted by the transmission channel. However, in the quantum domain both the QECC encoder and decoder circuitry impose more substantial imperfections, which simply cannot ignored. A further challenge is that we additionally have to deal with the specific quantum-domain phenomenon of error proliferation, because a single quantum-gate error encountered by a quantum encoder will in fact precipitate multiple component errors, rather than simply passing on its input errors without proliferating them. This motivates the design of inherently fault-tolerant quantum computation, which is capable of correcting both the self-inflicted errors imposed by its own encoder and decoder as well as the errors caused by the quantum channel.

V. CHALLENGES AND OPEN PROBLEMS

Quantum signal processing relies on delicate quantum particles, such as photons and electrons. Hence, any interaction with the surrounding environment will compromise the integrity of the desired operation. An immeasurable amount of effort has been invested in trying to minimize the presence of decoherence by perfecting the hardware implementation of the qubits as well as by developing sophisticated error correction procedures. Many of the QECC techniques are rooted in their classical counterparts [20]. However, to achieve an excellent error correction performance, long QECC codewords are required, which have to rely on a large number of qubits [21]–[23]. The problem with this approach is that at the time of writing most quantum circuits have a shorter coherence time than the time required for carrying out the decoding of long QECCs. Hence at the time of writing low-complexity yet powerful short codes are required for mitigating the effects of short coherence times.

Another aspect requiring substantial attention is to find meaningful applications, where the unique benefits quantum computing may be exploited, even of they have only a few hundred qubits. To elaborate a little further, quantum search, factoring and optimization problems tend to require thousands to millions of qubits. Another intriguing idea is to connect many medium-sized quantum computers with the aid of the quantum Internet relying on teleportation protocols for creating more powerful quantum computers. Some attractive applications are constituted by the variational quantum eigensolver (VQE) [24] and the quantum approximate optimization algorithm (QAOA) [25].

Finally, to fully realize the quantum Internet, a whole suite of quantum computers relying on superconducting, trapped ion, magnetic resonance, optical and other technologies have to be benchmarked. Furthermore, the entire gamut of quantum links, such as free space terrestrial, satellite, fiber optic and other connections will have to be further developed. Similarly, sophisticated protocols, such as for example, routing, multiple access, as well as repeat-and-request solutions will require massive standardization efforts.

Indeed, the road to the perfecty secure quantum communications era is inevitably a rocky one, which requires the collaboration of the entire IEEE community. This is why about half-a-dozen IEEE Societies have formed a New Initiative in Quantum Engineering (https:qce.quantum.ieee.org) the the new multi-disciplinary open access journal of quantum engineering (https:quantum.ieee.orgpublications)

Valued Colleague, we invite you to join this exhileratinng multi-disciplinary journey to solve some of the above-mentioned problems of true frontier-research into Communications V 2.0!

REFERENCES

[1] S. Kemp, "Digital 2020: Global Digital Overview," 2020.

[2] R. D. Van Meter III, *Architecture of a Quantum Multicomputer Optimized for Shors Factoring Algorithm.* PhD thesis, Keio University, 2006.

[3] C. H. Bennett and G. Brassard, "Quantum Cryptography: Public Key Distribution and Coin Tossing," in *Proceedings of the IEEE International Conference on Computers, Systems, and Signal Processing*, pp. 175–179, 1984.

[4] A. K. Ekert, "Quantum Cryptography Based on Bells Theorem," *Physical Review Letters*, vol. 67, no. 6, 1991.

[5] C. H. Bennett and S. J. Wiesner, "Communication via One- and Two-Particle Operators on Einstein-Podolsky-Rosen States," *Physical Review Letters*, vol. 69, no. 20, 1992.

[6] F.-G. Deng, G. L. Long, and X.-S. Liu, "Two-Step Quantum Direct Communication Protocol using the Einstein-Podolsky-Rosen Pair Block," *Physical Review A*, vol. 68, no. 4, 2003.

[7] D. Pan, K. Li, D. Ruan, S-X. Ng and L. Hanzo Single-photon-memory two-step quantum secure direct communication relying on Einstein-Podolsky-Rosen pairs, IEEE Access, 2020

[8] Z. Sun, LY. Song, Q. HUang, LG. Yin, GL. Long, JH. Lu and L. Hanzo: Towards Practical Quantum Secure Direct Communication: A Quantum-Memory-Free Protocol and Code Design, IEEE Transactions on Communications, Xplore Early access, 2020

[9] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, “Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels,” *Physical review letters*, vol. 70, no. 13, 1993.

[10] A. S. Cacciapuoti, M. Caleffi, R. Van Meter, and L. Hanzo, “When Entanglement Meets Classical Communications: Quantum Teleportation for the Quantum Internet,” *IEEE Transactions on Communications*, 2020.

[11] H. V. Nguyen, Z. Babar, D. Alanis, P. Botsinis, D. Chandra, M. A. M. Izhar, S. X. Ng, and L. Hanzo, “Towards the Quantum Internet: Generalised Quantum Network Coding for Large-Scale Quantum Communication Networks,” *IEEE Access*, vol. 5, pp. 17288–17308, 2017.

[12] R. Cleve and H. Buhrman, “Substituting Quantum Entanglement for Communication,” *Physical Review A*, vol. 56, no. 2, 1997.

[13] A. Broadbent, J. Fitzsimons, and E. Kashefi, “Universal Blind Quantum Computation,” in *50th Annual IEEE Symposium on Foundations of Computer Science (FOCS)*, pp. 517–526, IEEE, 2009.

[14] M. Hillery, V. Bužek, and A. Berthiaume, “Quantum Secret Sharing,” *Physical Review A*, vol. 59, no. 3, 1999.

[15] P. Botsinis, S. X. Ng, and L. Hanzo, “Quantum Search Algorithms, Quantum Wireless, and a Low-Complexity Maximum-Likelihood Iterative Quantum Multi-User Detector Design,” *IEEE access*, vol. 1, pp. 94–122, 2013.

[16] D. Alanis, P. Botsinis, Z. Babar, H. V. Nguyen, D. Chandra, S. X. Ng, and L. Hanzo, “A quantum-search-aided dynamic programming framework for pareto optimal routing in wireless multihop networks,” *IEEE Transactions on Communications*, vol. 66, no. 8, pp. 3485–3500, 2018.

[17] P. Botsinis, D. Alanis, Z. Babar, S. X. Ng, and L. Hanzo, “Noncoherent Quantum Multiple Symbol Differential Detection for Wireless Systems,” *IEEE Access*, vol. 3, pp. 569–598, 2015.

[18] P. Botsinis, D. Alanis, Z. Babar, S. X. Ng, and L. Hanzo, “Joint Quantum-Assisted Channel Estimation and Data Detection,” *IEEE Access*, vol. 4, pp. 7658–7681, 2016.

[19] P. Botsinis, D. Alanis, S. Feng, Z. Babar, H. V. Nguyen, D. Chandra, S. X. Ng, R. Zhang, and L. Hanzo, “Quantum-assisted indoor localization for uplink mm-wave and downlink visible light communication systems,” *IEEE Access*, vol. 5, pp. 23327–23351, 2017.

[20] Z. Babar, D. Chandra, H. V. Nguyen, P. Botsinis, D. Alanis, S. X. Ng, and L. Hanzo, “Duality of quantum and classical error correction codes: Design principles and examples,” *IEEE Communications Surveys & Tutorials*, vol. 21, no. 1, pp. 970–1010, 2018.

[21] Z. Babar, P. Botsinis, D. Alanis, S. X. Ng, and L. Hanzo, “The Road from Classical to Quantum Codes: A Hashing Bound Approaching Design Procedure,” *IEEE Access*, vol. 3, pp. 146–176, 2015.

[22] Z. Babar, P. Botsinis, D. Alanis, S. X. Ng, and L. Hanzo, “Fifteen Years of Quantum LDPC Coding and Improved Decoding Strategies,” *iEEE Access*, vol. 3, pp. 2492–2519, 2015.

[23] Z. Babar, Z. B. K. Egilmez, L. Xiang, D. Chandra, R. G. Maunder, S. X. Ng, and L. Hanzo, “Polar Codes and Their Quantum-Domain Counterparts,” *IEEE Communications Surveys & Tutorials*, vol. 22, no. 1, pp. 123–155, 2020.

[24] A. Peruzzo, J. McClean, P. Shadbolt, M.-H. Yung, X.-Q. Zhou, P. J. Love, A. Aspuru-Guzik, and J. L. Obrien, “A Variational Eigenvalue Solver on A Photonic Quantum Processor,” *Nature Communications*, vol. 5, p. 4213, 2014.

[25] E. Farhi, J. Goldstone, and S. Gutmann, “A Quantum Approximate Optimization Algorithm,” *arXiv preprint arXiv:1411.4028*, 2014.